# Using an equation for changing the angle between planes to find and evaluate planetary rings 

## Introduction

- NASA found a ring around an exoplanet(a planet not in the solar system)
- My supervisor, Dr Phil Sutton, had been running simulations and producing models for that ring with the goal of finding the ring's structure.
- I developed an equation which could change the angle between the $x-y$ plane.
- I used this equation along with other transformations to manipulate the ring so more data could be extrapolated and analysed.


## Results

- Using the equation I developed, there were some distinct 'bands' to the ring that could be seen with more clarity after the manipulation, which meant the particles could be analysed more clearly. One of these rings can be seen below (Original on left, transformed ring on the right.


Figure 2

- Other transformations that were tried yielded results where the ring was clearer and some aspects of the ring could be seen clearly. Hopefully they could be used in the final piece of the research to understand the final simulation of the ring.
- I concluded, from analysing the rings after performing the transformations, that there were some unusual actions in the from within the ring. One such ring had a thin, larger ring around the outside of the main ring. I assumed this was from something passing the ring and disturbing the particles.


## Conclusion

- My supervisor and I found some interesting transformed rings that seemed to show potential for further understanding of the rings structure.
- I saw how the world of academic research worked and how a academic's work is developed over time.
- I found new skills and managed to deal with software I had never used before, using it all from home during the Coronavirus outbreak.
- I discovered a passion for coding and astrophysics that I hope to use in my third year and in my future career.


## References

Figures 1 and 2 produced in SPLASH on Ubuntu.
Figure 3 produced in Desmos. https://www.desmos.com/calculator

## Method

- First, I installed Ubuntu, GDL - a coding software that worked well for simulating planetary rings - and SPLASH. SPLASH was useful for visualizing a snapshot of the ring and displaying the manipulations and changes that I applied to the code.
- Once familiarized, I applied my equation and others (taking logarithms of desired variables and trying to plot the ring in polar coordinates.
- I worked through several different snapshots of the ring, comparing the unmanipulated ring with the manipulated version.
- Finally, I analyzed the manipulated rings and tried to build a picture up of what was happening to the ring at certain points.
- An example of a manipulated ring can be seen below. The left image was the original and the right was the transformed ring. You can see clearly that the transformation has resulted in a diagonally elongated ring where some particles can be seen with more clarity



Figure 1

## How the equation worked

- I developed the equation by manipulating the matrix for rotating a point in a 2D space.
- The matrix for rotations (anticlockwise) are:

$$
\left[\begin{array}{cc}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{array}\right]
$$

- Using some basic mathematics you can achieve a pair of parametric equations for the new $x$ and y co-ordinate, $x^{\prime} \& y^{\prime}$, where $\theta$ is the change in angle between the 2 planes.

$$
\begin{aligned}
& x^{\prime}=x \cos \theta+y \sin \theta \\
& y^{\prime}=y \cos \theta-x \sin \theta
\end{aligned}
$$

- Since I was rotating the y-plane and left the x-plane alone, you can allow $x^{\prime}=x$ and place that into the equation for $y^{\prime}$. As y is just a function of x , it is possible to change the equation to:

$$
y^{\prime}=f(x) \cos \theta-x \sin \theta
$$

- Below is a graphical representation of what this equation calculates. The red line is the original function, the broken red line is the new function. The broken black line is where the new $y$-axis is positioned after the angle change occurs.


